Ex: 8.1

Q1. Use Euclid's division algorithm to find the HCF of :

(i) 135 and 225 $(225 > 135)$

Sol:- According to Euclids division lemma

(a, b) $[a > b]$ then $\Rightarrow$ $a = bq + r$

$\therefore a = 225$ $(a > b)$

$b = 135$

$225 = (135)(1) + 90$

$135 = (90)(1) + 45$

$90 = (\underline{45})(2) + 00$

$\therefore HCF(135, 225) = \underline{45}$

135) 225 (1
135
90

90) 135 (1
90
45

45) 90 (2
90
00

Ex: 8.1

Q1. Use Euclid's division algorithm to find the HCF of :

(ii) 196 and 38220

Sol:- According to Euclids div. lemma

$38220 > 196$

$(a) > (b)$

Now, $a = bq + r$

$$\therefore 38220 = \underbrace{196}_{HCF} \times 195 + 0$$

$$\begin{array}{r|l}
196 \,\big)\, 38220 \,\big(\, 195 & \\
196\downarrow & \\
\hline
1862 & \\
1764\downarrow & \\
\hline
980 & \\
980 & \\
\hline
000 &
\end{array}$$

$\therefore$ HCF of 196 and 38220 is 196

Ex: 8.1

Q1. Use Euclid's division algorithm to find the HCF of :
(iii) 867 and 255

Sol:- According to Euclids division lemma

$$a = bq + r \quad (a > b)$$

$$\therefore a = 867 , \ b = 255 \quad (867 > 255)$$

$$867 = 255 \times 3 + 2$$

$$255 = 2 \times 127 + 1$$

$$2 = \underset{\text{HCF}}{\underset{\downarrow}{\underline{1}}} \times 2 + 0$$

$\therefore$ HCF of 867 and 255 is 1

255)867(3
865
2

2)255(127
2
05
4
15
14
1

Ex: 8.1

Q2. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

Let us start with taking a

where a is a positive odd integer.

According to Euclids Division Lemma; $a = bq + r$, $\mathbf{0 \leq r < b}$.

We apply the division algorithm with a and $b = 4$.

Since $0 \leq r < 5$, the possible remainders are 0, 1, 2, 3, 4, 5

That is, a can be 6q, or 6q + 1, or 6q + 2, or 6q + 3, or 6q + 4, or 6q + 5

where q is the quotient. However, since a is odd, a cannot be 6q or 6q + 2 or 6q + 4

How? And Why?

$6q = 2(3q) = 2k_1$

$6q + 2 = 2(3q + 1) = 2k_2$

$6q + 4 = 2(3q + 2) = 2k_3$, which are even numbers.

Therefore, any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$. Hence Proved.

min. ⇒ LCM

Ex: 8.1

HCF

Q3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Acc. to EDL (a, b) $a > b$

$a = bq + r$

$616 = (32)(19) + 8$

$32 = (8)(4) + 0$

$\therefore$ HCF$(616, 32) = 8$

The max. no. of col. in which they can march = 8 col.

$$\begin{array}{r|l} & 19 \\ 32 & 616 \\ & 32 \\ \hline & 296 \\ & 288 \\ \hline & 008 \end{array}$$

$32 \times 9 = 288$

Ex: 8.1

Q4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m

Let us start with taking a

where a is a positive odd integer.

According to Euclids Division Lemma; $a = bq + r$, $\mathbf{0 \leq r < b}$.

We apply the division algorithm with a and $b = 3$.

Since $0 \leq r < 3$, the possible remainders are 0, 1, 2

That is, a can be $3q$, or $3q + 1$, or $3q + 2$

(i) $a = 3q$ $(r = 0)$

Squaring Both sides

$a^2 = (3q)^2$

$a^2 = 9q^2$

$a^2 = 3(3q^2)$

Let, $3q^2 = m$

So, $a^2 = 3m$

(i) $a = 3q + 1$, $(r = 1)$

Squaring Both sides

$a^2 = (3q + 1)^2$

$a^2 = 9q^2 + 1 + 6q$

$a^2 = 3(3q^2 + 6q) + 1$

Let, $3q^2 + 6q = m$

So, $a^2 = 3m + 1$

(i) $a = 3q + 2$, $(r = 2)$

Squaring Both sides

$a^2 = (3q + 2)^2$

$a^2 = 9q^2 + 4 + 6q$

$a^2 = 3(3q^2 + 6q + 1) + 1$

Let, $3q^2 + 6q + 1 = m$

So, $a^2 = 3m + 1$

Ex: 8.1

Q5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8

Let us start with taking a

where a is a positive odd integer.

According to Euclids Division Lemma; $a = bq + r$, $\mathbf{0 \leq r < b}$.

We apply the division algorithm with a and $b = 9$.

Since $0 \leq r < 9$, the possible remainders are 0, 1, 2, 3, 4, 5, 6, 7, 8.

That is, a can be $9q$, or $9q + 1$, or $9q + 2$.

(i) $a = 9q \ (r = 0)$

Cubing on Both sides

$a^3 = (9q)^3$

$a^3 = 729q^3$

$a^3 = 9(81q^3)$

Let, $81q^3 = m$

So, $a^3 = 9m$

(ii) $a = 9q + 1 \ (r = 1)$

Cubing on Both sides

$a^3 = (9q + 1)^3$

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$a^3 = 729q^3 + 1 + 243q^2 + 27q$

$a^3 = 9(81q^3 + 27q^2 + 3q) + 1$

Let, $81q^3 + 27q^2 + 3q = m$

So, $a^3 = 9m + 1$

(iii) $a = 9q + 2 \ (r = 2)$

Cubing on Both sides

$a^3 = (9q + 2)^3$

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$a^3 = 729q^3 + 8 + 486q^2 + 108q$

$a^3 = 9(81q^3 + 54q^2 + 12q) + 8$

Let, $81q^3 + 54q^2 + 12q = m$

So, $a^3 = 9m + 8$